Gravitational waves from primordial black hole fluctuations

Theodoros Papanikolaou

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Introduction

- Primordial Black Holes (PBHs) form in the early universe out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta \equiv \frac{\delta\rho}{\rho_{\mathrm{b}}} > \delta_{\mathrm{c}}(w \equiv p/\rho)$ [Carr - 1975].

$$m_{\mathrm{PBH}} = \gamma M_{\mathrm{H}} \propto H^{-1} \text{ where } \gamma \sim \mathrm{O}(1)$$

Introduction

- Primordial Black Holes (PBHs) form in the early universe out of the **collapse of enhanced energy density perturbations** upon horizon reentry of the typical size of the collapsing overdensity region. This happens when $\delta \equiv \dfrac{\delta\rho}{\rho_{\rm b}} > \delta_{\rm c}(w \equiv p/\rho)$ [Carr - 1975].

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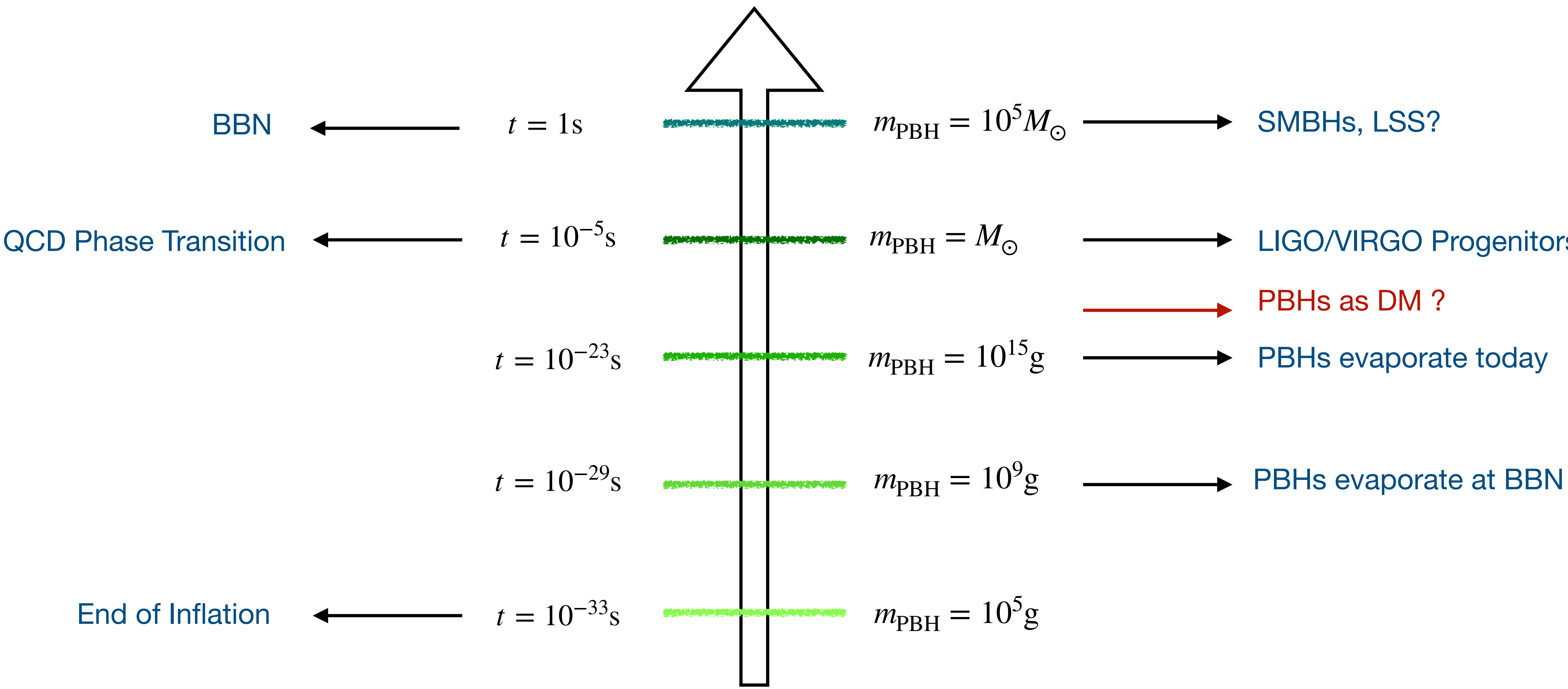

See for reviews in [Carr et al.- 2020, Sasaki et al - 2018, Clesse et al. - 2017]

PBHs and GWs

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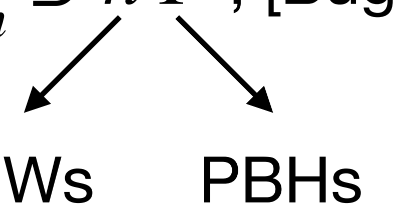

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- 4) **GWs induced** at second order **by PBH energy density fluctuations** [Papanikolaou et al. - 2020], abundantly produced during a PBH-dominated era.

PBH-dominated era phenomenology

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- Evaporation of light PBHs can also **produce naturally the baryon asymmetry** through CP violating out-of-equilibrium decays of Hawking evaporation products [J. D. Barrow et al. - 1991, T. C. Gehrman et al. - 2022, N. Bhaumik et al. - 2022].

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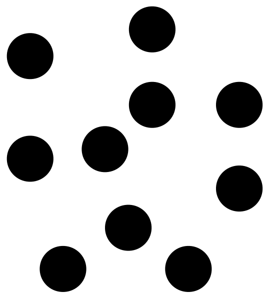
- **GWs induced by PBH energy density fluctuations can interpret** in a very good agreement **the recently released PTA GW data** [Lewicki et al. - 2023, Basilakos et al. - 2023]

Gravitational waves from PBH fluctuations: The Gaussian case

[**T. Papanikolaou**, V. Vennin, D. Langlois, JCAP 03 (2021) 053]

The PBH Matter Field



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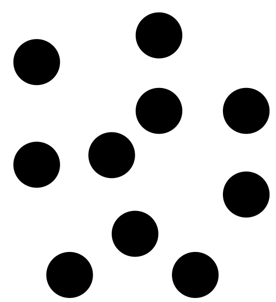

Poisson Statistics [Desjacques & Riotto - 2018, Ali-Haimoud - 2018]

Same mass [Dizgah, Franciolini & Riotto - 2019]

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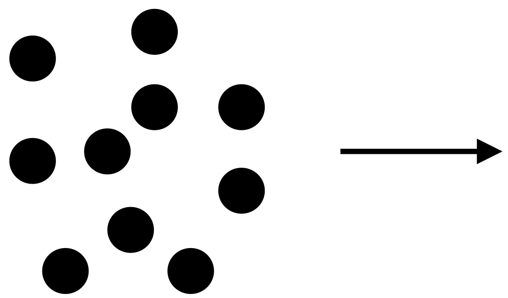

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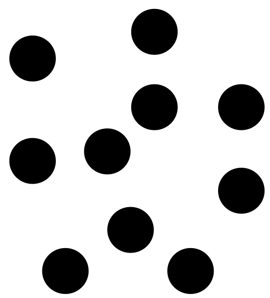

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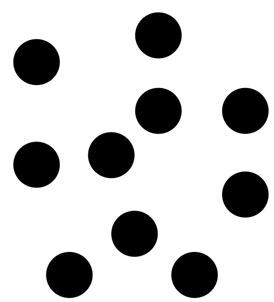

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[Isocurvature perturbation]

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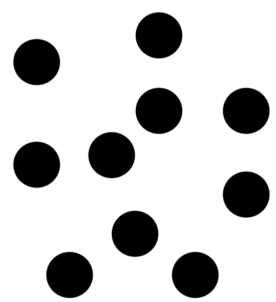

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This **isocurvature perturbation,** $\delta_{\rm PBH}$ generated during the RD era **will convert** during the PBHD era **to a curvature perturbation** $\zeta_{\rm PBH}$, associated to a PBH gravitational potential Φ.

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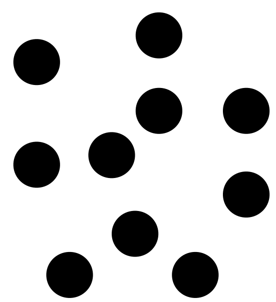

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$$\mathscr{P}_\Phi(k) = S_\Phi^2(k) \frac{2}{3\pi} \left(\frac{k}{k_{\rm UV}} \right)^3 \left(5 + \frac{4}{9} \frac{k^2}{k_{\rm d}^2} \right)^{-2}, \text{ with } S_\Phi(k) \equiv \left(\sqrt{\frac{2}{3}} \frac{k}{k_{\rm evap}} \right)^{-1/3}$$

Basics of Scalar Induced Gravitational Waves

- Choosing as the gauge for the GW frame the Newtonian gauge, the metric is written as

$$\mathrm{d}s^2 = a^2(\eta)\left\{-(1+2\Phi)\mathrm{d}\eta^2 + \left[(1-2\Phi)\delta_{ij} + \frac{h_{ij}}{2}\right]\mathrm{d}x^i\mathrm{d}x^j\right\}.$$

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$$S^s_{\vec{k}} = \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^{3/2}} e^s_{ij}(\vec{k})q_i q_j \left[2\Phi_{\vec{q}}\Phi_{\vec{k}-\vec{q}} + \frac{4}{3(1+w)}(\mathcal{H}^{-1}\Phi'_{\vec{q}} + \Phi_{\vec{q}})(\mathcal{H}^{-1}\Phi'_{\vec{k}-\vec{q}} + \Phi_{\vec{k}-\vec{q}})\right].$$

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- Considering here only sub-horizon scales, where a flat spacetime approximation can be applied, the energy density of GWs reads as [M. Maggiore - 2000]

$$\rho_{\mathrm{GW}}(\eta,\vec{x}) = \frac{M^2_{\mathrm{Pl}}}{8}\overline{\left(\partial_t h_{\alpha\beta}\partial_t h^{\alpha\beta} + \partial_i h_{\alpha\beta}\partial_i h^{\alpha\beta}\right)}.$$

The Gravitational Wave Spectrum

- The spectral abundance, $\Omega_{\rm GW}(\eta, k)$ of GWs can be written as:

$$\Omega_{\rm GW}(\eta, k) \equiv \frac{1}{\rho_{\rm tot}} \frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln k} = \frac{1}{24} \left(\frac{k}{a(\eta)H(\eta)} \right)^2 \mathcal{P}_h(\eta, k)$$

$$\text{with} \quad \mathcal{P}_h(\eta, k) \equiv \frac{k^3 |h_k|^2}{2\pi^2} \propto \int {\rm d}v \int {\rm d}u \left(\int f(v, u, k, \eta) {\rm d}\eta \right)^2 \mathcal{P}_\Phi(kv) \mathcal{P}_\Phi(ku) \,.$$

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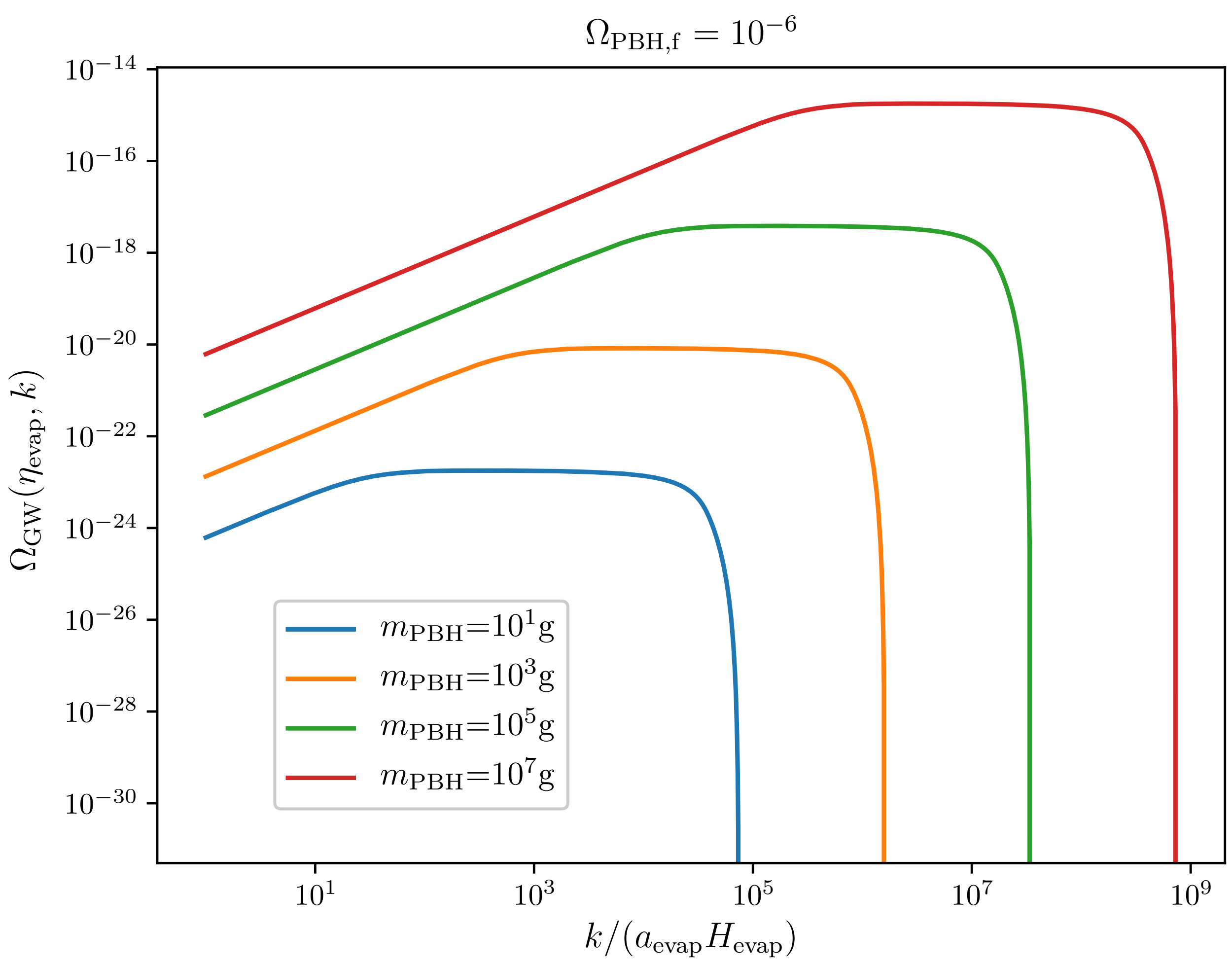

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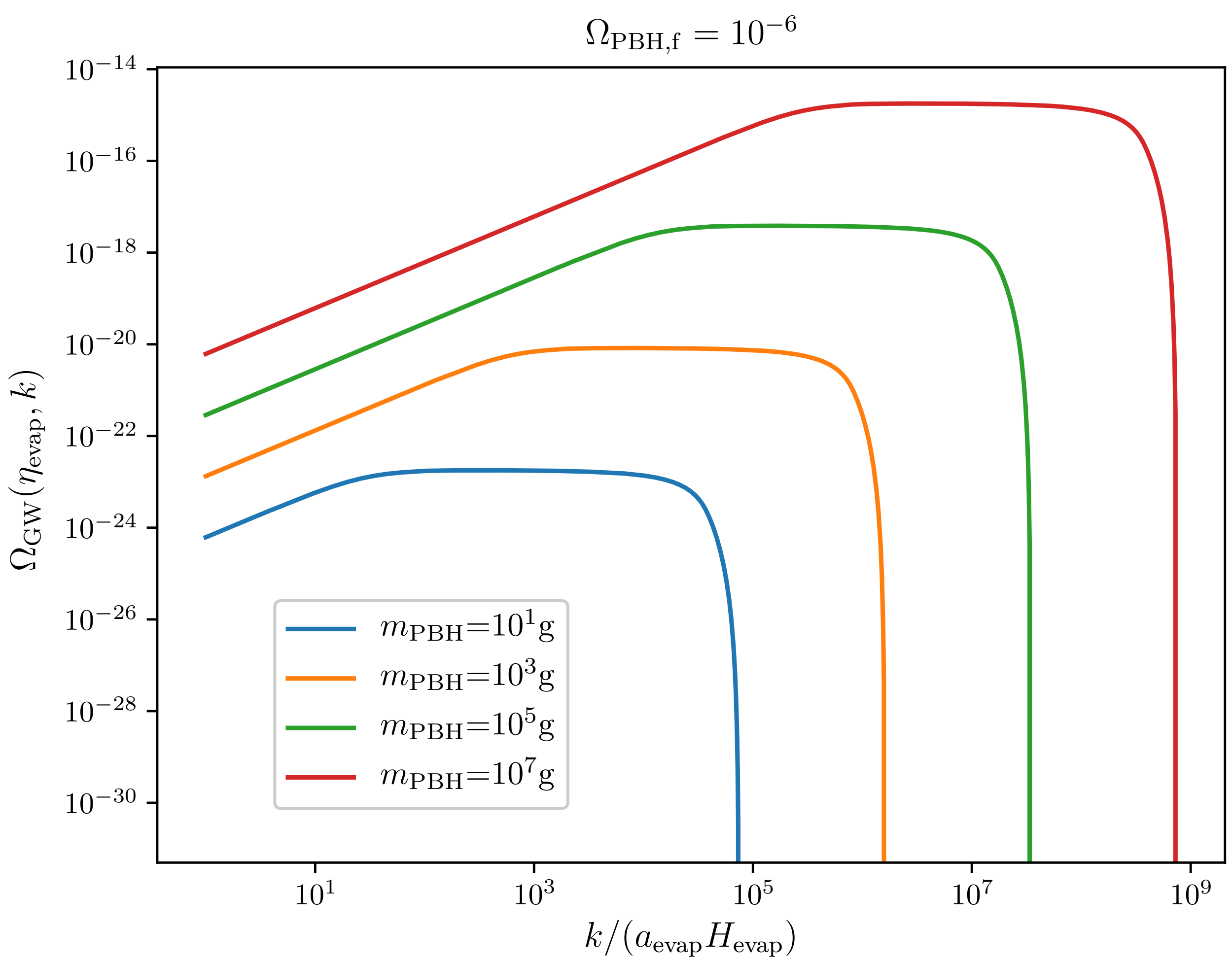

$$\Omega_{\rm GW}(\eta_{\rm evap}, k) \propto \left(\frac{m_{\rm PBH}}{10^9 {\rm g}} \right)^{4/3} \Omega_{\rm PBH,f}^{16/3} \times \begin{cases} \frac{k}{k_{\rm d}} & \text{for} \quad k \ll \mathscr{H}_{\rm d} \\ 8 & \text{for} \quad k \gg \mathscr{H}_{\rm d} \end{cases}$$

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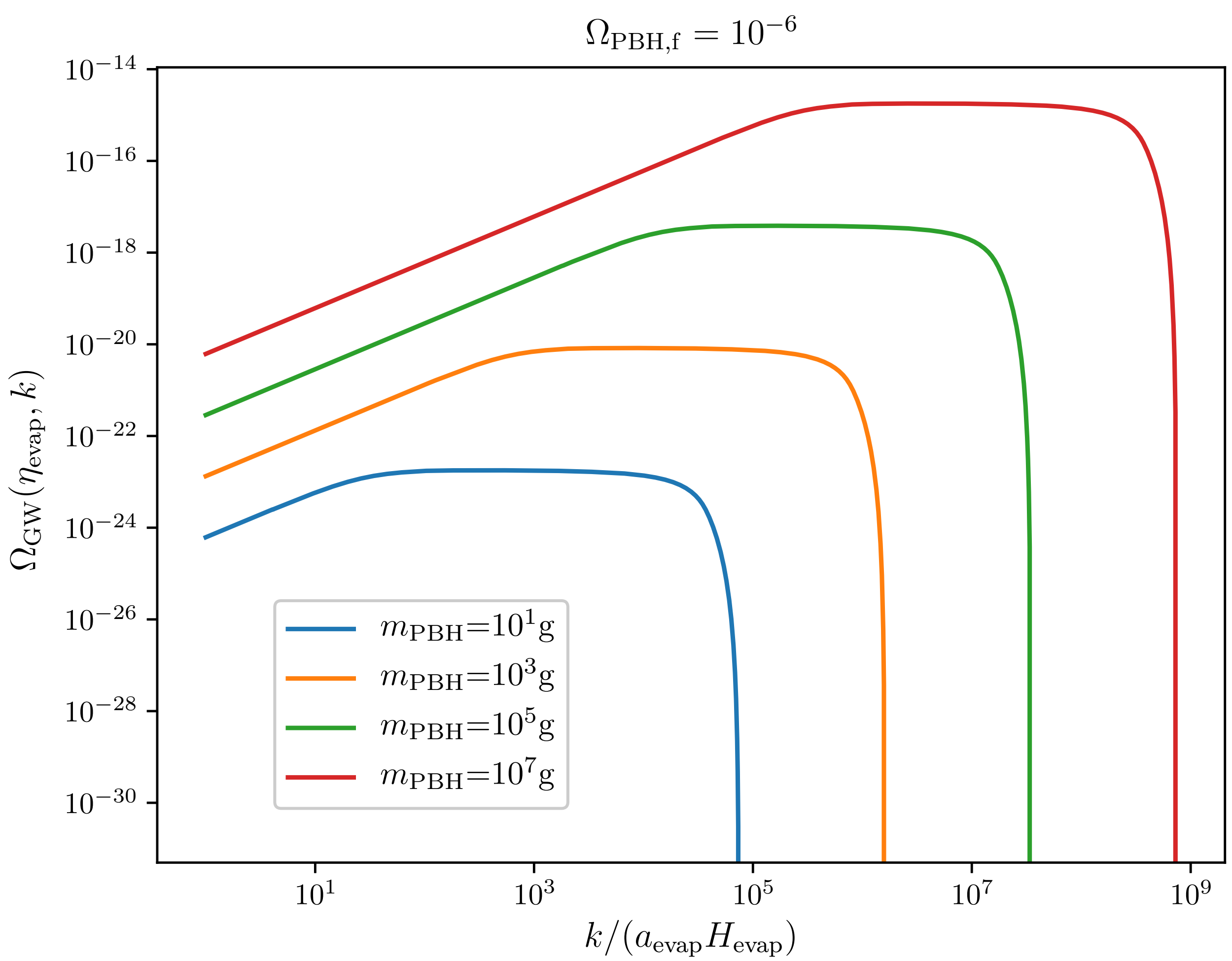

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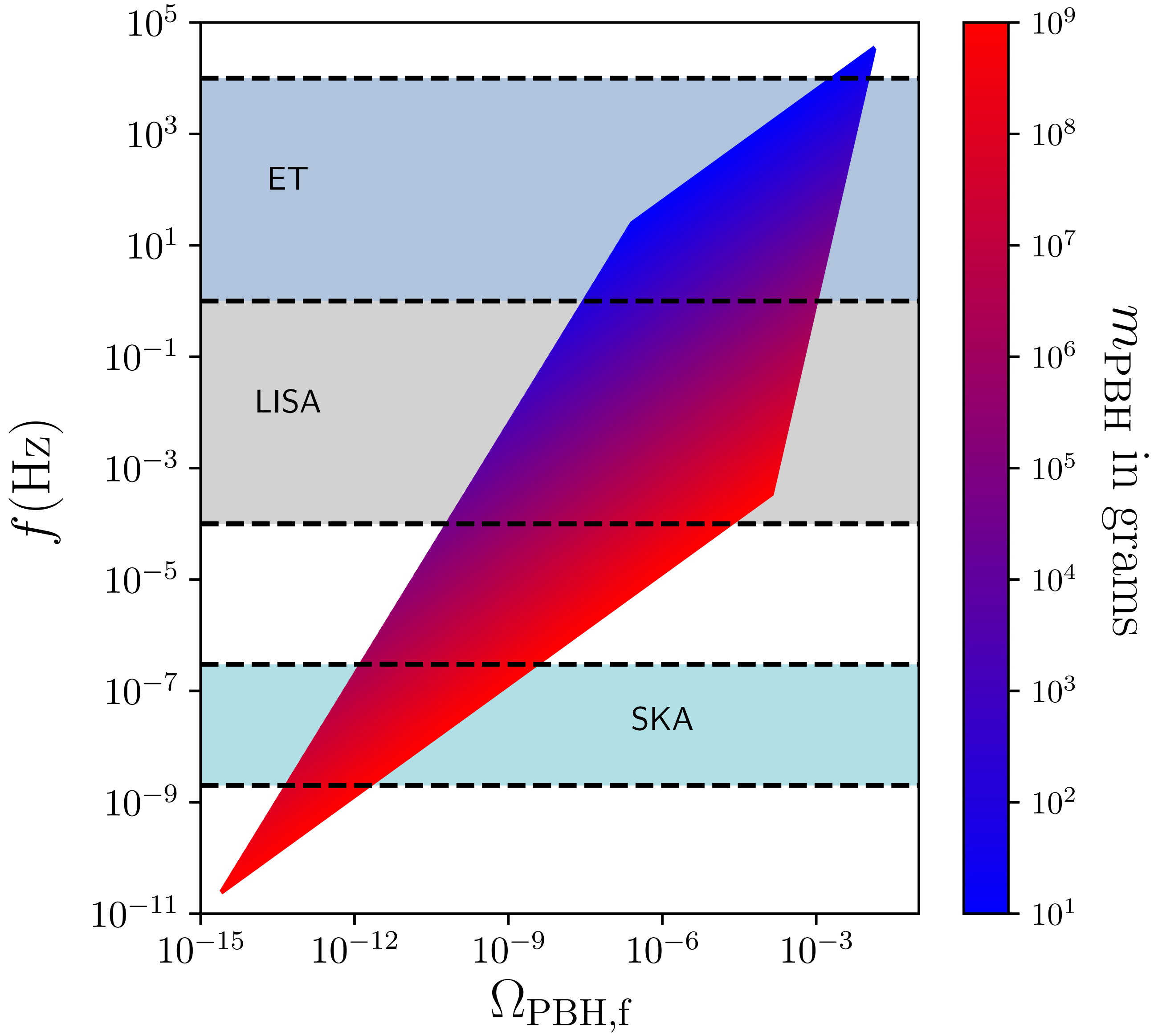
$$\Omega_{\rm GW,tot}(\eta_{\rm evap}) \leq 1 \Rightarrow \Omega_{\rm PBH,f} \leq 10^{-4}\left(\frac{10^9\mathrm{g}}{m_{\rm PBH}}\right)^{1/4}$$

[T.Papanikolaou, V. Vennin, D. Langlois - 2020]

GW Detectability



[Papanikolaou et al. - 2020]

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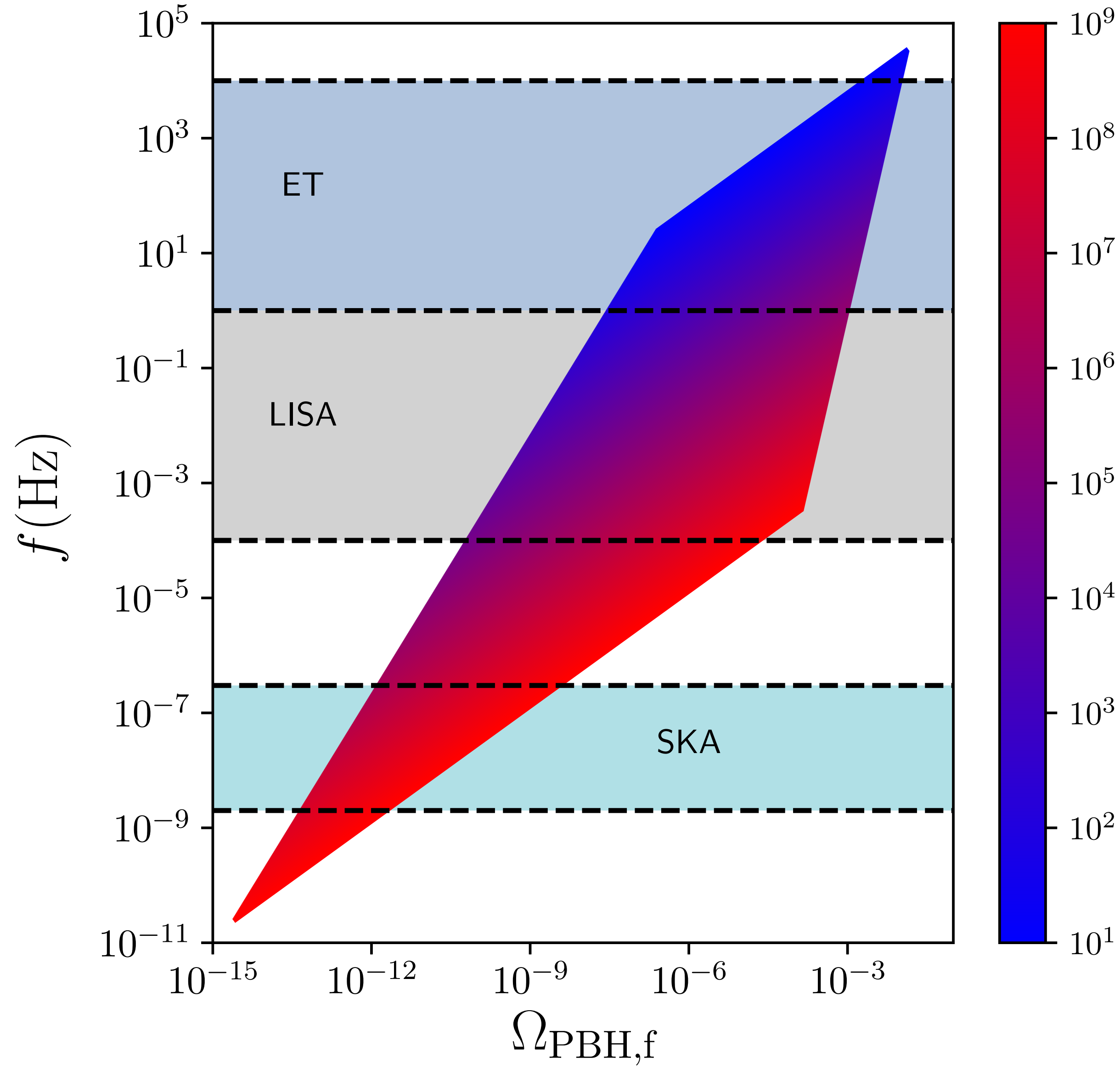

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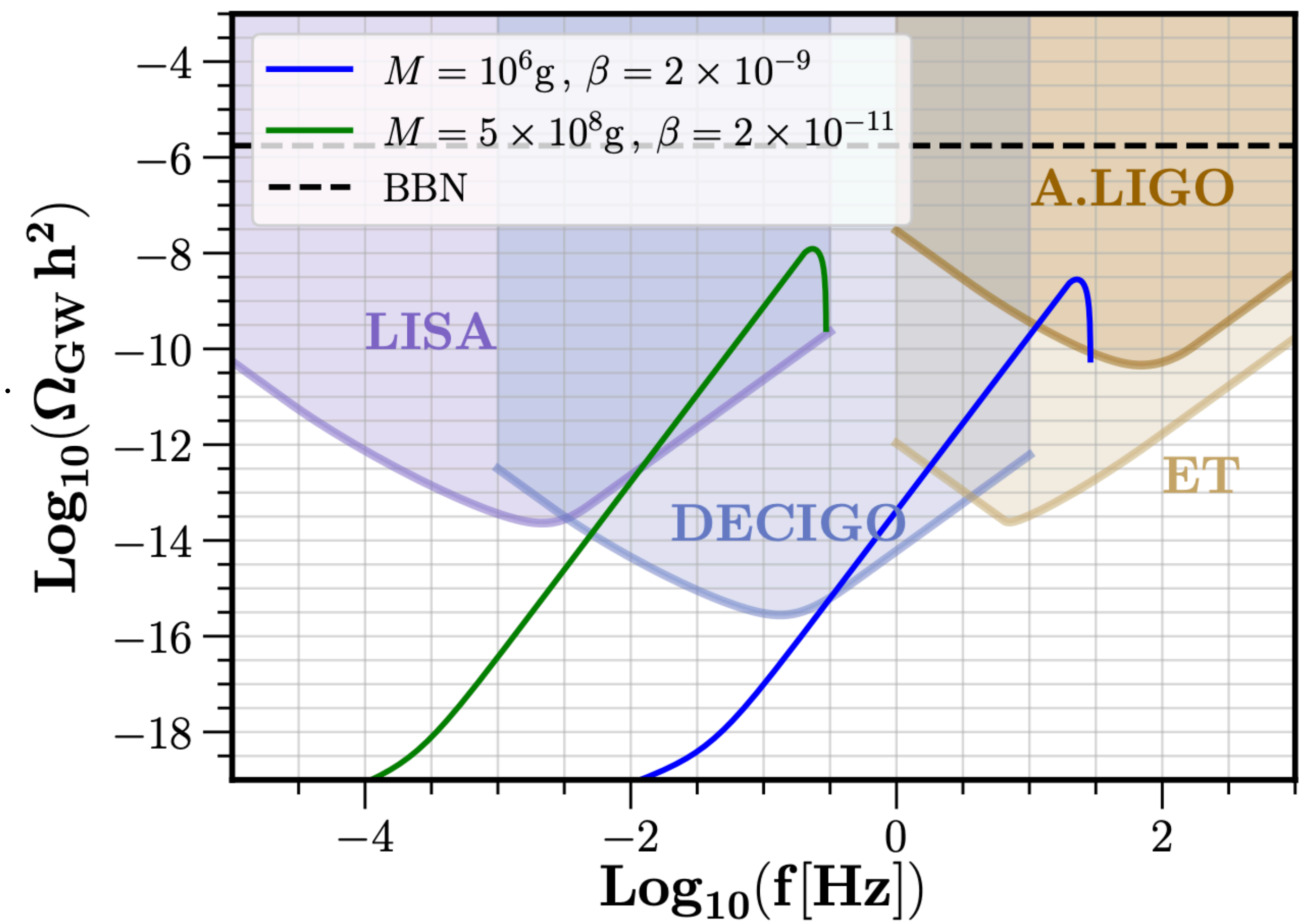

[Domenech et al. - 2020]

- **GWs induced by a dominating gas of PBHs might be detectable** in the future with gravitational-waves experiments.
- In the case of a monochromatic PBH mass distribution one finds a **sudden transition between the PBH-dominated and the radiation-dominated era** [Inomata et al. - 2019, Domenech et al. - 2020].

The effect of an extended PBH mass distribution

[**T. Papanikolaou**, JCAP 10 (2022) 089]

The PBH mass function and the PBH abundance

$$\mathscr{P}_\zeta(k) = A_\zeta \left(k/k_0\right)^{n_\mathrm{s}(k)-1},$$

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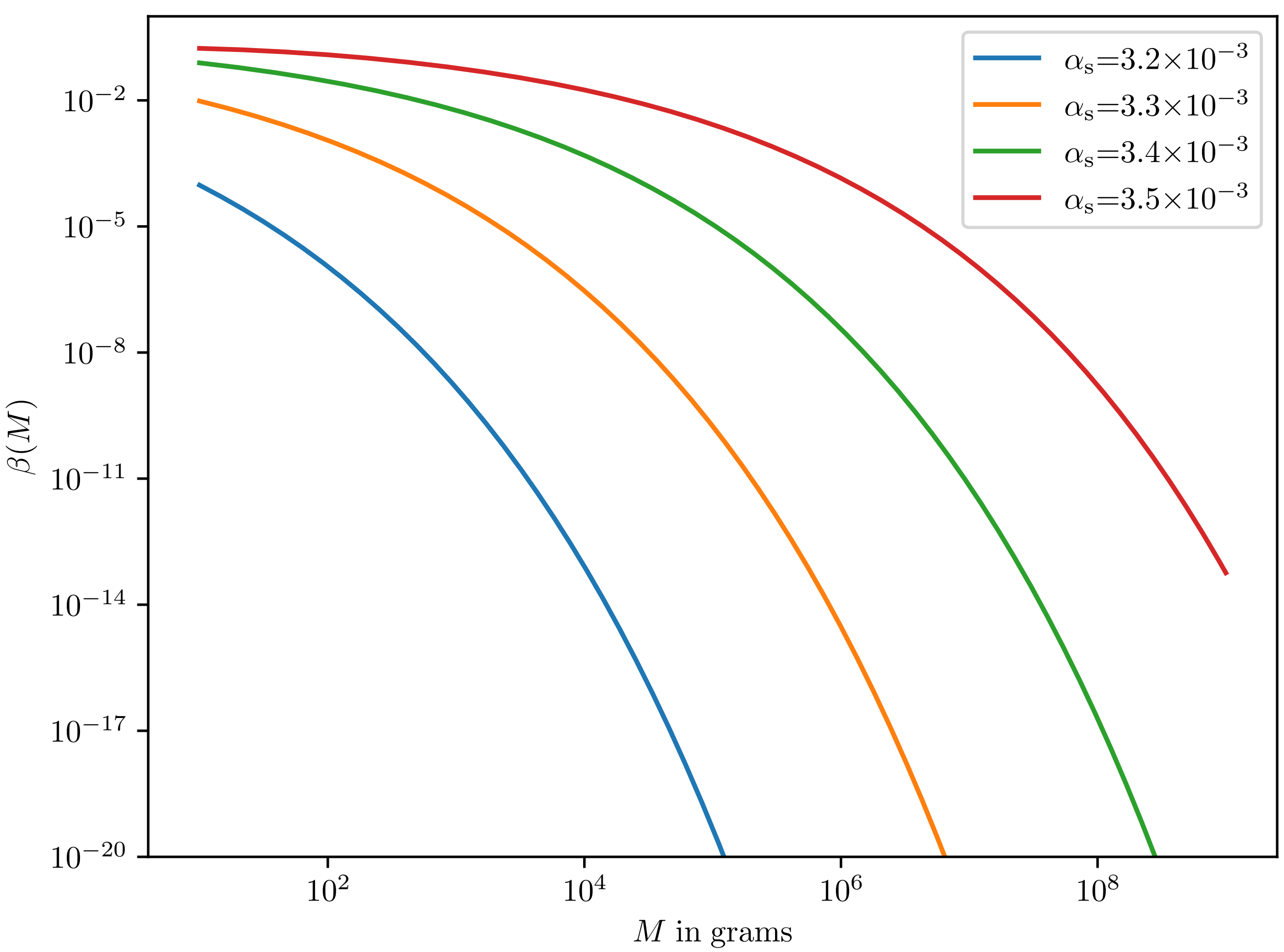

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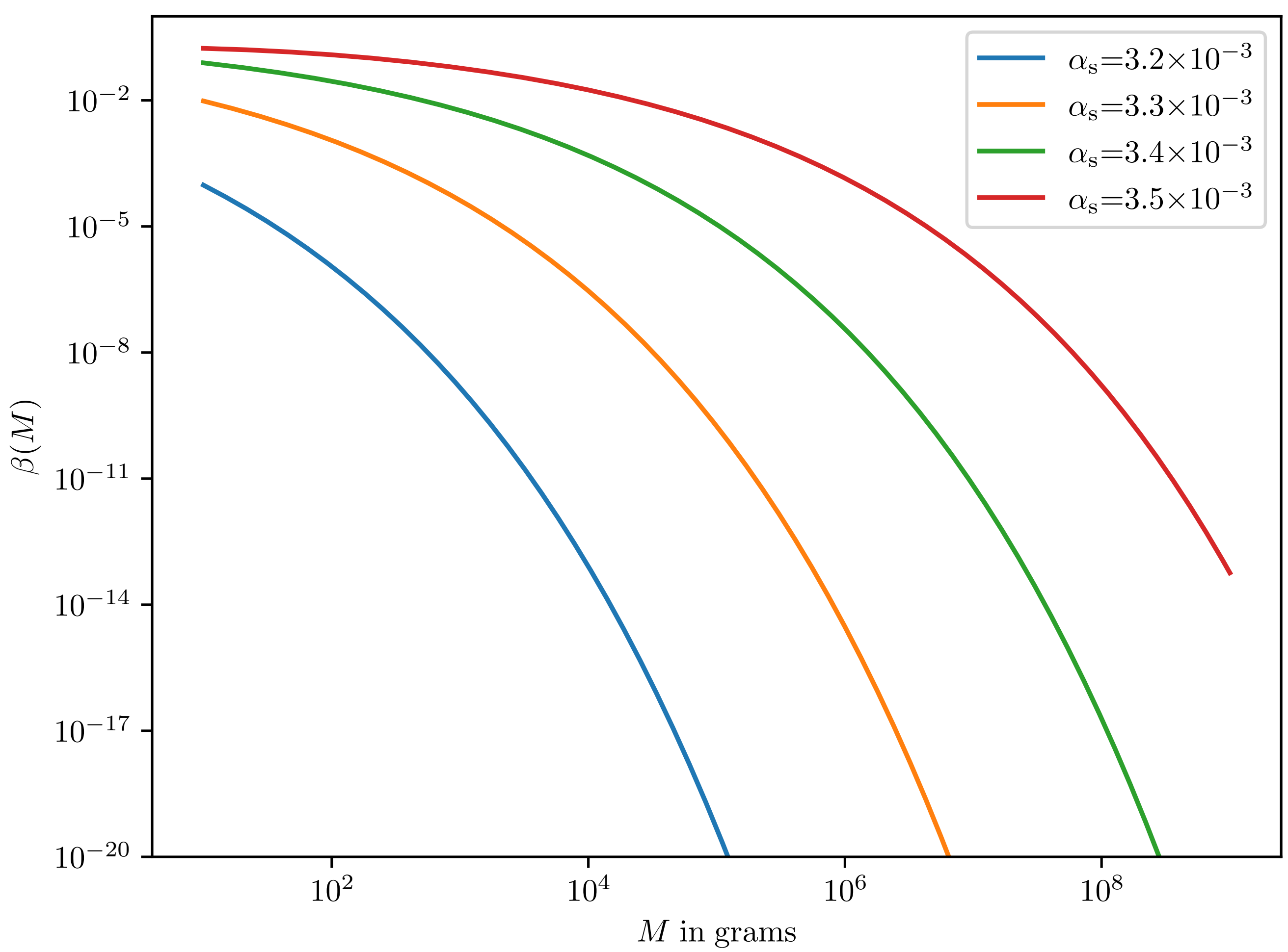
$$\beta(M) \equiv \frac{1}{\rho_{tot}}\frac{d\rho_{PBH}}{d\ln M} \text{ within peak theory}$$

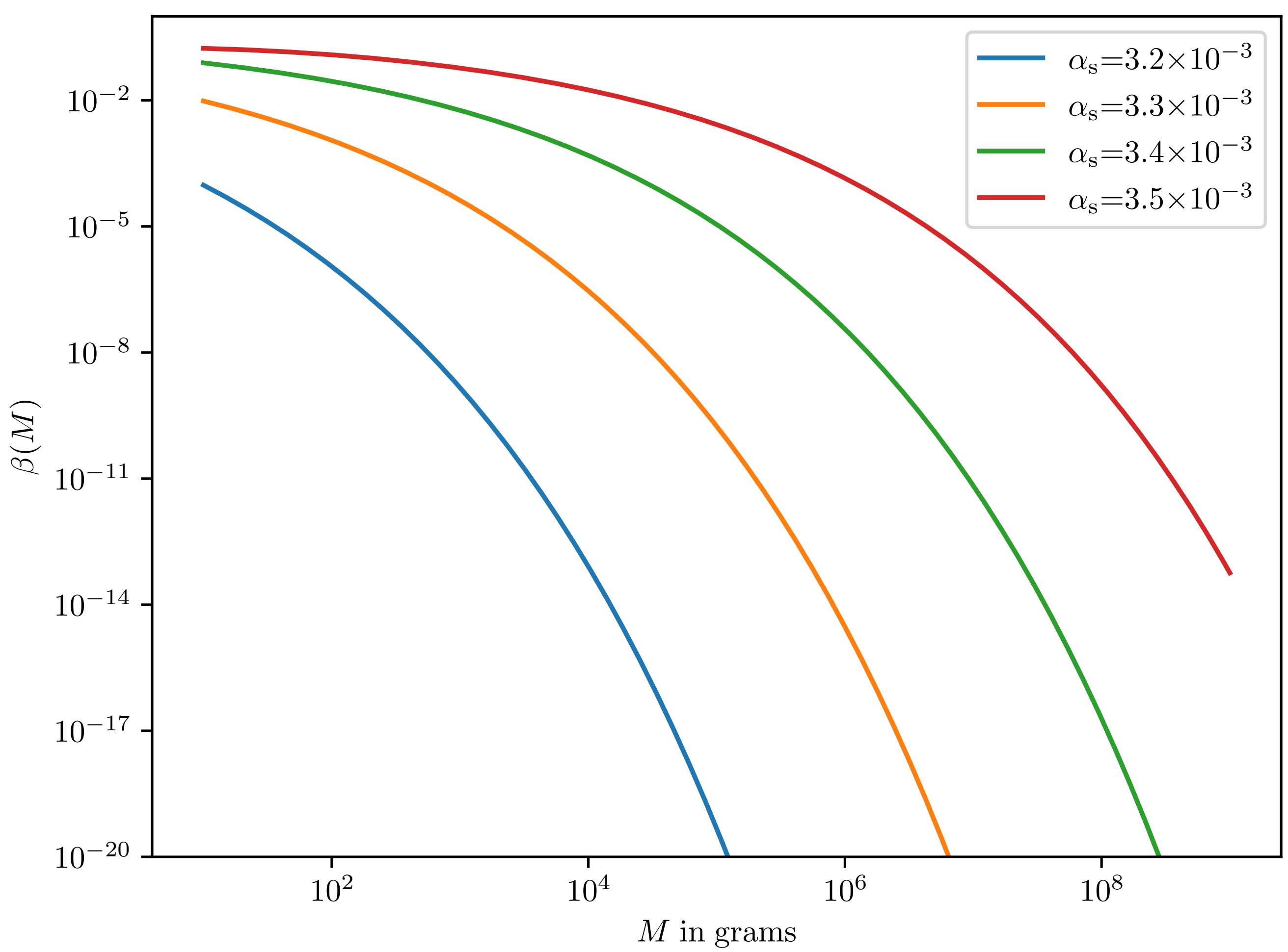
$$\Omega_{PBH}(t) = \int_{M_{min}}^{M_{max}} \bar{\beta}\,(M,t)\left\{1 - \frac{t - t_{ini}}{\Delta t_{evap}(M_f)}\right\}^{1/3} d\ln M$$

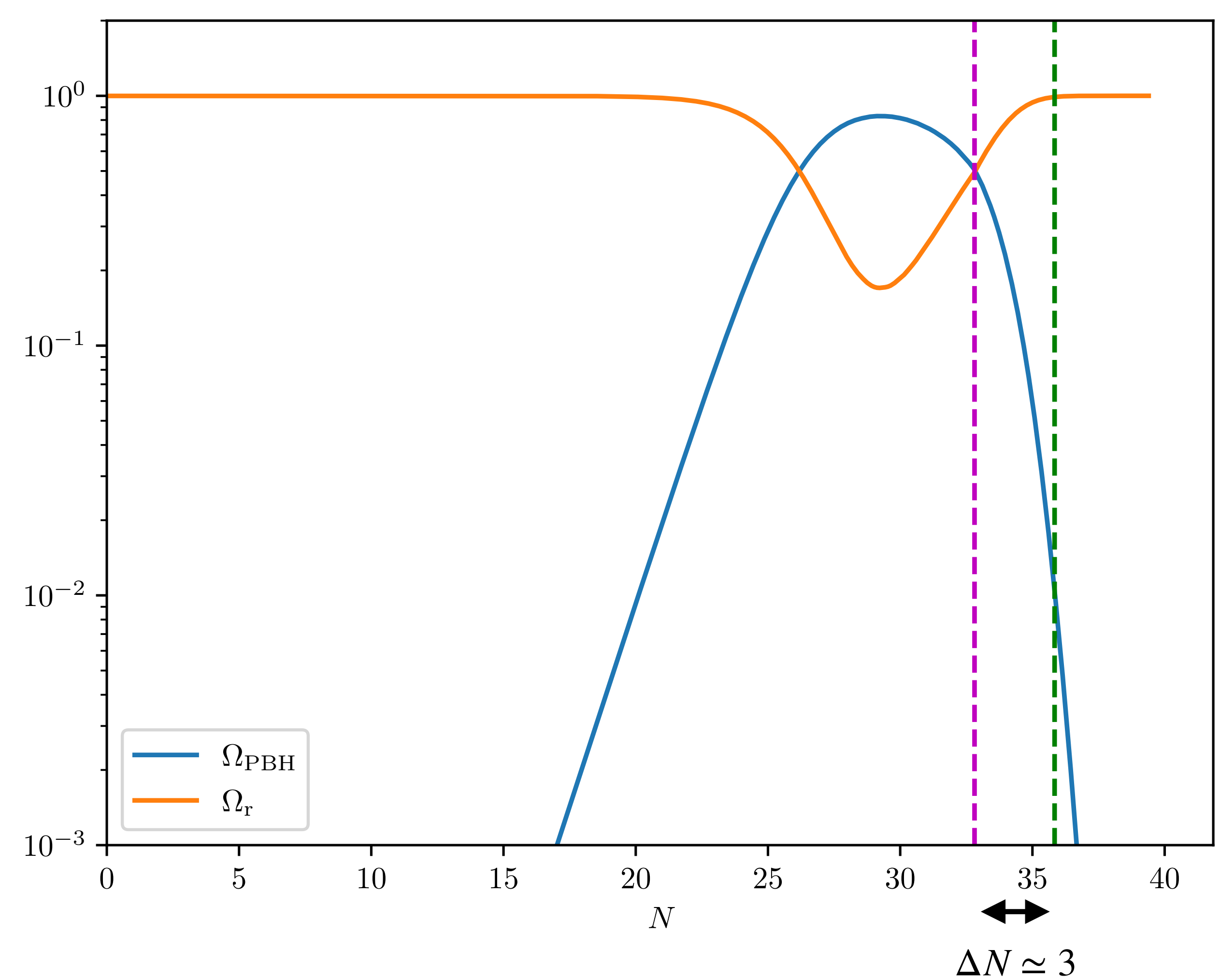


The PBH mass function and the PBH abundance

$$\mathscr{P}_\zeta(k) = A_\zeta \left(k/k_0\right)^{n_s(k)-1},$$
$$\text{with } n_s(k) = n_{s,0} + \frac{\alpha_s}{2!}\ln\left(\frac{k}{k_0}\right)$$

$$\beta(M) \equiv \frac{1}{\rho_{\rm tot}}\frac{d\rho_{\rm PBH}}{d\ln M} \text{ within peak theory}$$
$$\Omega_{\rm PBH}(t) = \int_{M_{\rm min}}^{M_{\rm max}} \bar{\beta}(M,t)\left\{1 - \frac{t - t_{\rm ini}}{\Delta t_{\rm evap}(M_{\rm f})}\right\}^{1/3} d\ln M$$





Gradual Transition

The PBH matter power spectrum

- In this case, we have a gas of **PBHs with different masses.** We should define a PBH mean separation scale accounting for the extended PBH mass distribution function.

$$\langle M \rangle(t) \equiv \frac{\int_{M_{\rm min}}^{M_{\rm max}} M\bar{\beta}(M,t) \left\{ 1 - \frac{t - t_{\rm ini}}{\Delta t_{\rm evap}(M_{\rm f})} \right\}^{1/3} \mathrm{d}\ln M}{\int_{M_{\rm min}}^{M_{\rm max}} \bar{\beta}(M,t)\, \mathrm{d}\ln M} \Rightarrow \bar{r} = \left(\frac{3\langle M \rangle}{4\pi\rho_{\rm PBH}} \right)^{1/3}.$$

$$P_{\delta_{\rm PBH}}(k) \equiv \langle |\, \delta_k^{\rm PBH} |^2 \rangle = \frac{4\pi}{3k_{\rm UV}^3}, \text{ where } k < k_{\rm UV} = \frac{a}{\bar{r}}$$

$$\mathscr{P}_\Phi(k) = S_\Phi^2(k) \frac{2}{3\pi} \left(\frac{k}{k_{\rm UV}} \right)^3 \left(5 + \frac{4}{9}\frac{k^2}{k_{\rm d}^2} \right)^{-2}$$

Evolving the PBH gravitational potential

- Our physical system is comprised by **matter in form of PBHs** which "decays" to **radiation** through the process of PBH evaporation.

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$$\delta'_{\rm PBH} = -\theta_{\rm PBH} + 3\Phi' - a\Gamma\Phi$$

$$\theta'_{\rm PBH} = -\mathscr{H}\theta_{\rm PBH} + k^2\Phi$$

$$\delta'_{\rm r} = -\frac{4}{3}(\theta_{\rm r} - 3\Phi') + a\Gamma\frac{\rho_{\rm PBH}}{\rho_{\rm r}}(\delta_{\rm PBH} - \delta_{\rm r} + \Phi)$$

$$\theta'_{\rm r} = \frac{k^2}{4}\delta_{\rm r} + k^2\Phi - a\Gamma\frac{3\rho_{\rm PBH}}{4\rho_{\rm r}}\left(\frac{4}{3}\theta_{\rm r} - \theta_{\rm PBH}\right)$$

$$\Phi' = -\frac{k^2\Phi + 3\mathscr{H}^2\Phi + \frac{3}{2}\mathscr{H}^2\left(\frac{\rho_{\rm PBH}}{\rho_{\rm tot}}\delta_{\rm PBH} + \frac{\rho_{\rm r}}{\rho_{\rm tot}}\delta_{\rm r}\right)}{3\mathscr{H}}$$

$$\delta_\alpha \equiv (\rho_\alpha - \rho_{\rm tot})/\rho_{\rm tot},$$

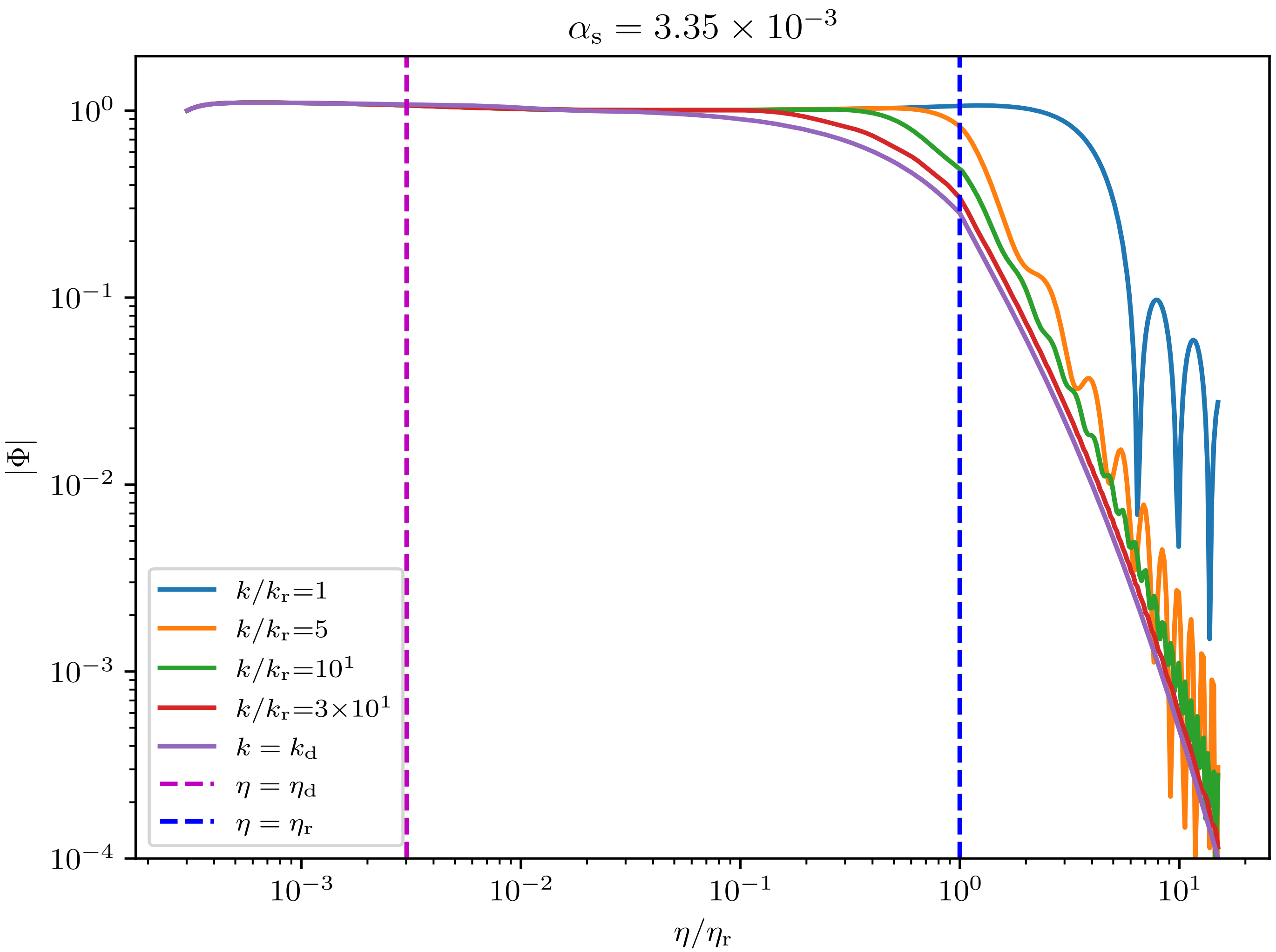
$$\theta \equiv \partial v_i/\partial x_i$$

$$' \equiv \frac{\rm d}{{\rm d}\eta}, \text{ with } {\rm d}\eta \equiv {\rm d}t/a$$

$$\langle\Gamma\rangle(t) = \frac{\int_{t_{\rm evap,min}}^{t_{\rm evap,max}}\beta(t_{\rm evap})\Gamma_M(t_{\rm evap}, t){\rm d}\ln t_{\rm evap}}{\int_{t_{\rm evap,min}}^{t_{\rm evap,max}}\beta(t_{\rm evap}){\rm d}\ln t_{\rm evap}}, \text{ with } \Gamma_M(t_{\rm evap}, t) \equiv -\frac{1}{M}\frac{{\rm d}M}{{\rm d}t} = \frac{1}{3(t_{\rm evap} - t)}$$

$$\text{Adiabatic initial conditions}: \delta_{\rm PBH,ini} = -2\Phi_{\rm ini}, \quad \delta_{\rm r,ini} = \frac{4}{3}\delta_{\rm PBH,ini}, \quad \theta_{\rm PBH,ini} = \theta_{\rm r,ini} = 0, \quad \Phi_{\rm ini} = 1$$

The gravitational potential Φ



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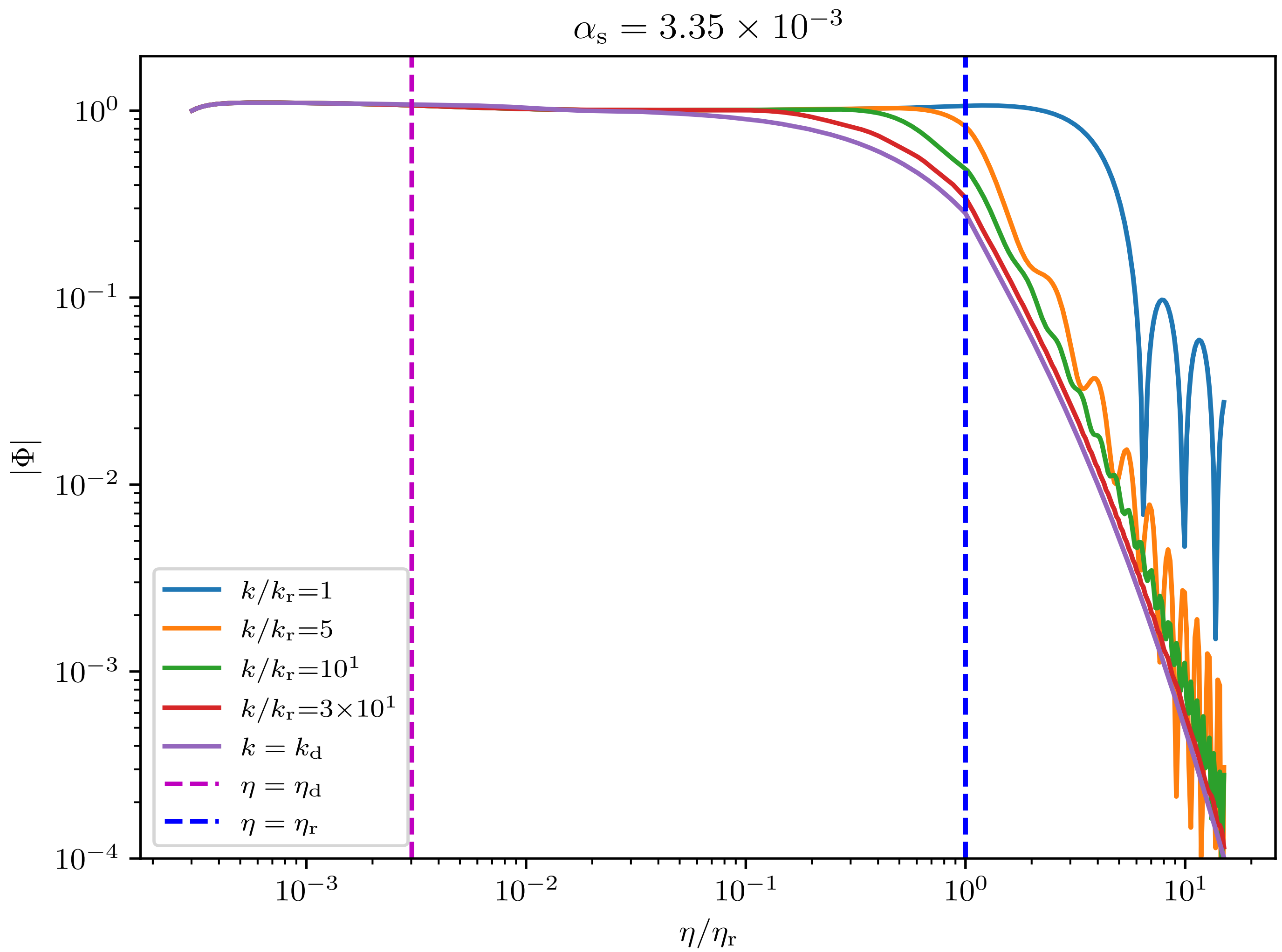

The scales considered

$$a)\ \delta_{\mathrm{PBH},k} \propto a : \delta_{\mathrm{PBH},k_{\mathrm{NL}}}(\eta_r) = 1 \Rightarrow k_{\mathrm{NL}} = k_{\mathrm{UV}}^{3/7}\left(\frac{3\pi}{2}\right)^{1/7}\left(\frac{a_d}{a_r}\right)^{2/7}\left(\frac{4a_d^2}{9t_d^2}\right)^{2/7}$$

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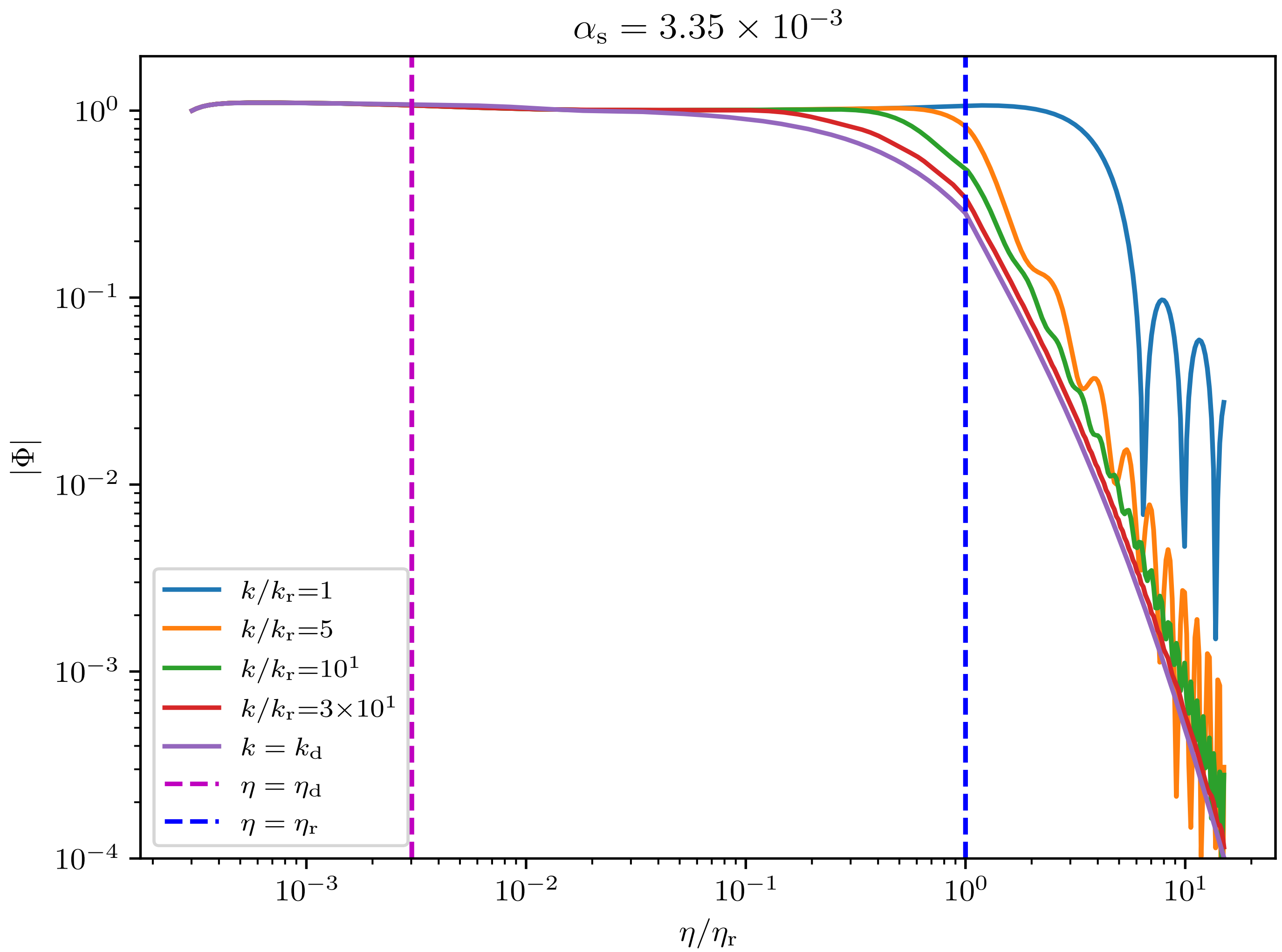

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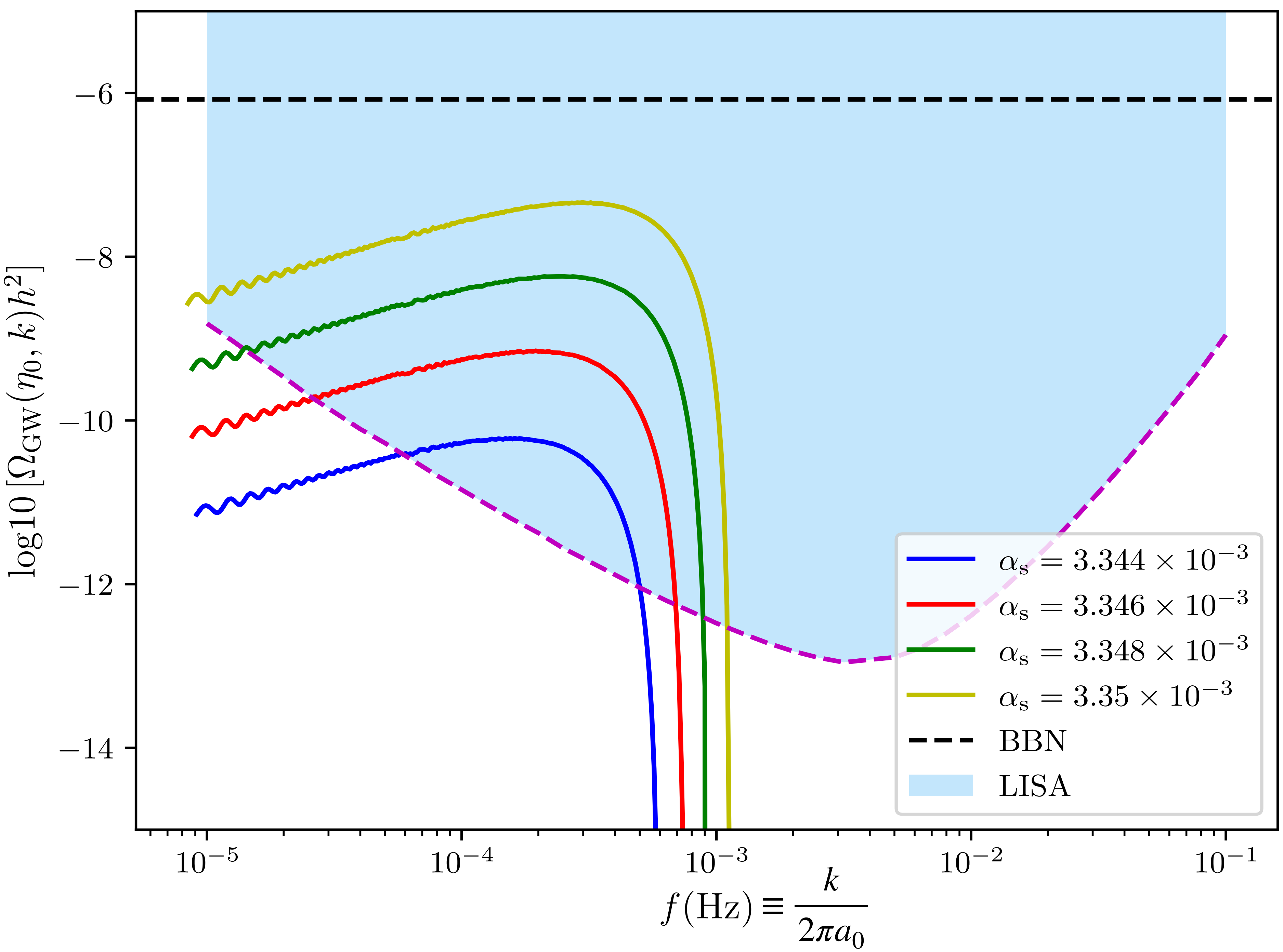
$b)$ Being quite conservative, we consider only modes $k \in [k_r, k_{\max}]$.

$$k_{\max} = \min[k_d, k_{\mathrm{NL}}] \Rightarrow \mathscr{P}_\Phi(k) = \frac{2}{75\pi}\left(\frac{k}{k_{\mathrm{UV}}}\right)^3$$

The GW spectrum



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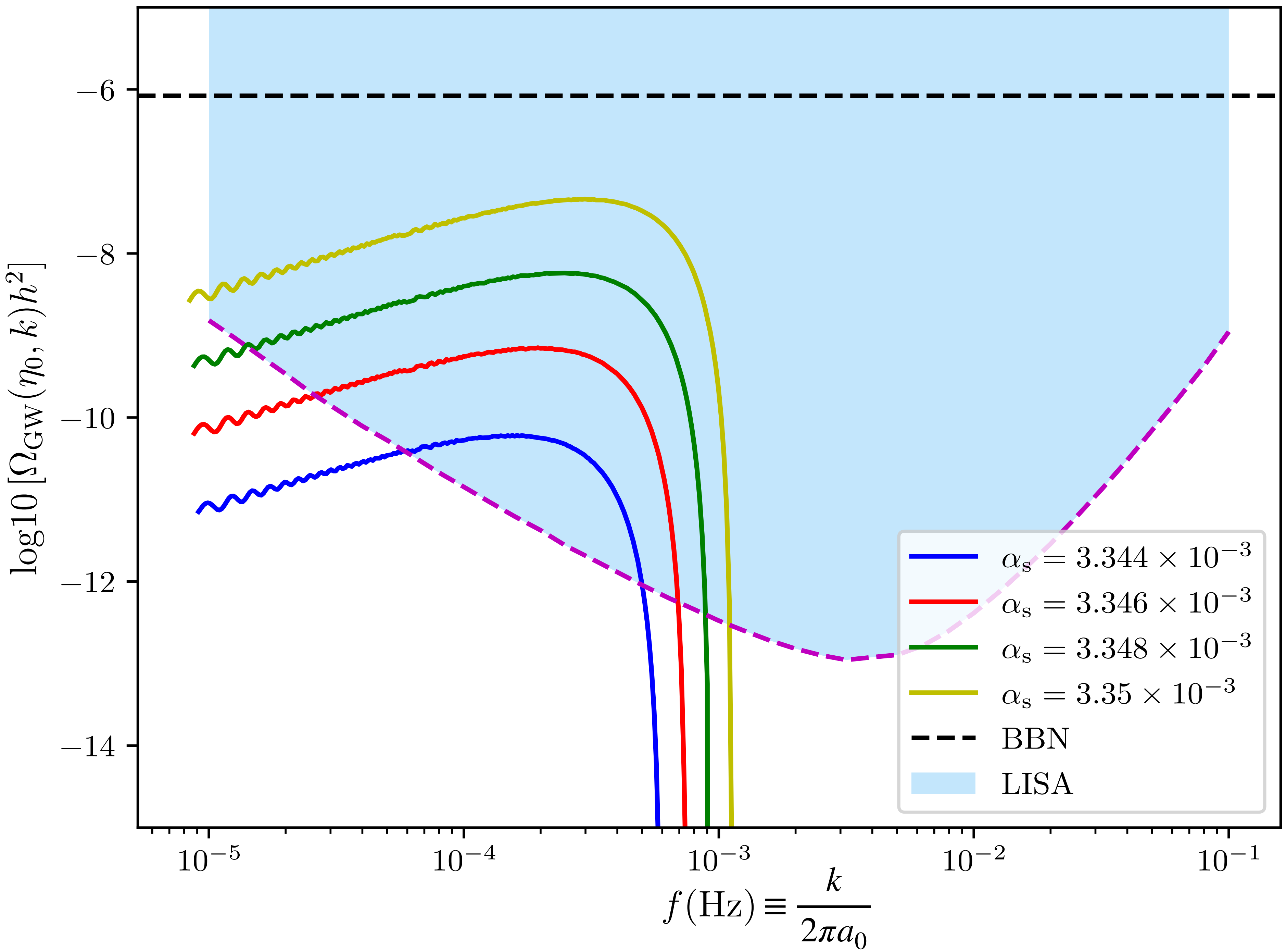

$$\left.\begin{array}{l} \eta_r > \eta_d \Rightarrow \alpha_s > 3.316 \times 10^{-3} \\ \Omega_{\rm GW,BBN} < 0.05 \Rightarrow \alpha_s < 3.355 \times 10^{-3} \end{array}\right\} \Rightarrow \boxed{3.316 \times 10^{-3} < \alpha_s < 3.355 \times 10^{-3}}$$

Gravitational waves from primordial black hole fluctuations: The effect of non-Gaussianities

[**T. Papanikolaou**, X. C. He, X. H. Ma, Y. F. Cai, E. N. Saridakis, M. Sasaki, 2403.00660]

Primordial non-Gaussianities of local type

$$\begin{aligned}
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2})P_{\mathcal{R}}(k) \\
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}) \\
&\quad \times \frac{6}{5} f_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2) + 2 \text{ perms}\right] \\
\langle \mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\mathcal{R}(\boldsymbol{k_4})\rangle &\equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}+\boldsymbol{k_4}) \\
&\quad \times \left\{ \frac{54}{25} g_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(k_3) + 3 \text{ perms}\right] \right. \\
&\qquad \left. + \tau_{\rm NL}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(|\boldsymbol{k_1}+\boldsymbol{k_3}|) + 11 \text{ perms}\right] \right\}
\end{aligned}$$

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$$\langle\mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\rangle \equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2})P_{\mathcal{R}}(k)$$

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$$\langle\mathcal{R}(\boldsymbol{k_1})\mathcal{R}(\boldsymbol{k_2})\mathcal{R}(\boldsymbol{k_3})\mathcal{R}(\boldsymbol{k_4})\rangle \equiv (2\pi)^3\delta^{(3)}(\boldsymbol{k_1}+\boldsymbol{k_2}+\boldsymbol{k_3}+\boldsymbol{k_4}) \times \left\{\frac{54}{25}g_{\mathrm{NL}}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(k_3)+3\text{ perms}\right] + \tau_{\mathrm{NL}}\left[P_{\mathcal{R}}(k_1)P_{\mathcal{R}}(k_2)P_{\mathcal{R}}(|\boldsymbol{k_1}+\boldsymbol{k_3}|)+11\text{ perms}\right]\right\}$$

[Path integral formalism for n-point correlation functions (galaxy halo bias)]

↓

[S. Matarrese et al. - 1986, S. Matarresse and L. Verde - 2008]

$$\xi_{\mathrm{PBH}}(\mathbf{x}_1,\mathbf{x}_2) \equiv \langle\delta_{\mathrm{PBH}}(\mathbf{x}_1)\delta_{\mathrm{PBH}}(\mathbf{x}_2)\rangle = \int \mathscr{P}_{\mathrm{PBH}}(k)e^{\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}\mathrm{d}\ln k$$

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$kR \ll 1$ $\quad$ $R \sim 1/k_{\rm f}$

$$\mathscr{P}_{\delta_{\rm PBH}}(k) \simeq \mathscr{P}_{\mathscr{R}}(k)\nu^4 \left(\frac{4}{9\sigma_R}\right)^4 \int \frac{\mathrm{d}^3p_1 \mathrm{d}^3p_2}{(2\pi)^6} \tau_{\rm NL}(p_1, p_2, p_1, p_2) W^2_{\rm local}(p_1) W^2_{\rm local}(p_2) P_{\mathscr{R}}(p_1) P_{\mathscr{R}}(p_2)$$

$$+\frac{k^3}{2\pi^2}(k - \text{independent terms})$$

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$$\mathscr{P}_{\Phi}(k) = S_\Phi^2(k)\left(5+\frac{4}{9}\frac{k^2}{k_{\rm d}^2}\right)^{-2}\left[\left(\frac{4\nu}{9\sigma_R}\right)^4\bar{\tau}_{\rm NL}\mathscr{P}_{\mathscr{R}}(k) + \mathscr{P}_{\delta_{\rm PBH},{\rm Poisson}}(k)\right]$$

The non-Gaussian PBH matter power spectrum

$$\text{Ansatz 1}: \mathscr{P}_{\mathscr{R}} = \mathscr{P}_{\mathscr{R}}(k_{\rm f}) e^{-\frac{1}{2\sigma^2}\ln^2\left(\frac{k}{k_{\rm f}}\right)} + 2.2 \times 10^{-9}\left(\frac{k}{0.05\mathrm{Mpc}^{-1}}\right)^{0.965-1}, \text{ with } \mathscr{P}_{\mathscr{R}}(\mathrm{k_f}) \simeq 10^{-2}$$

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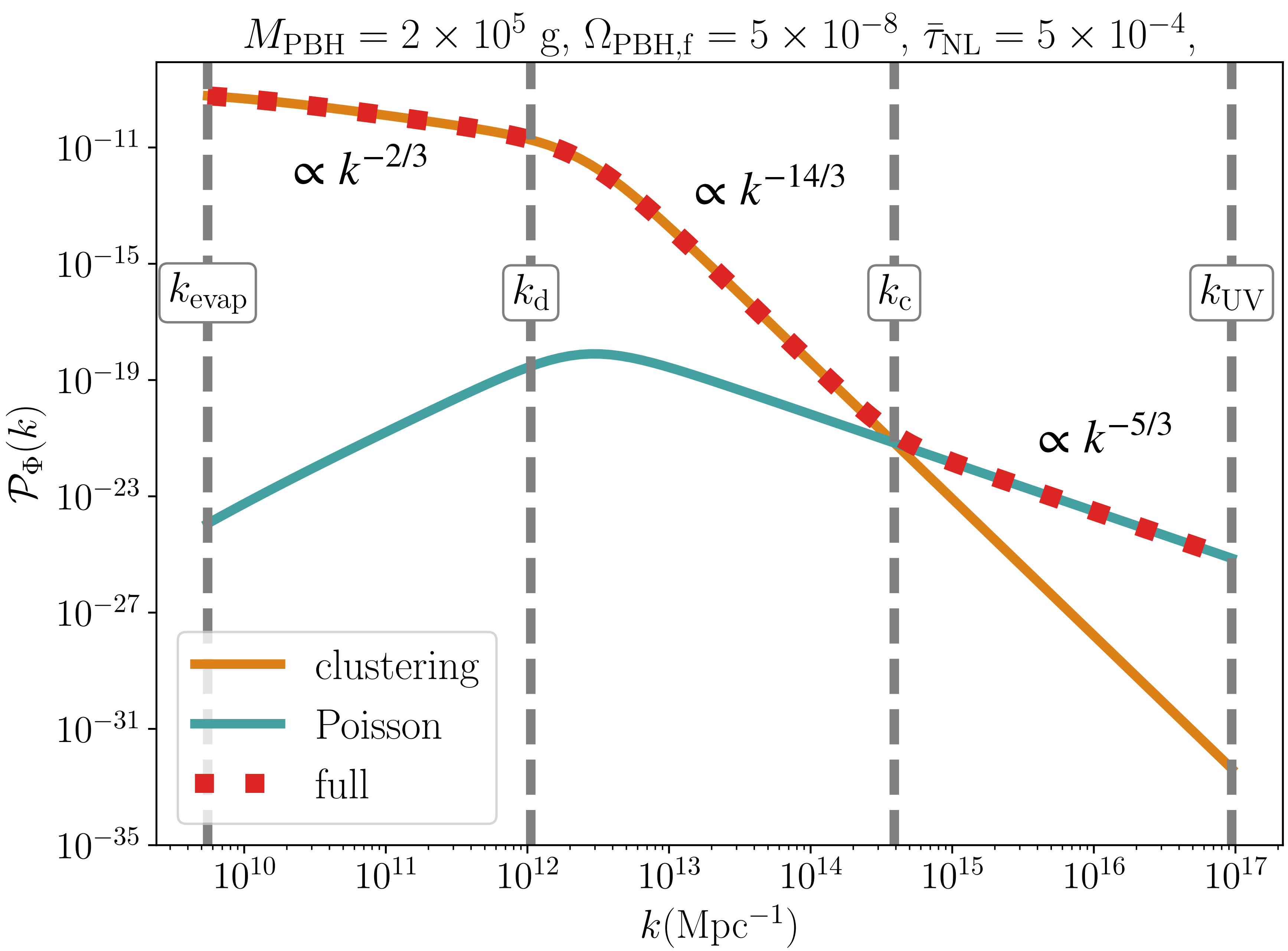

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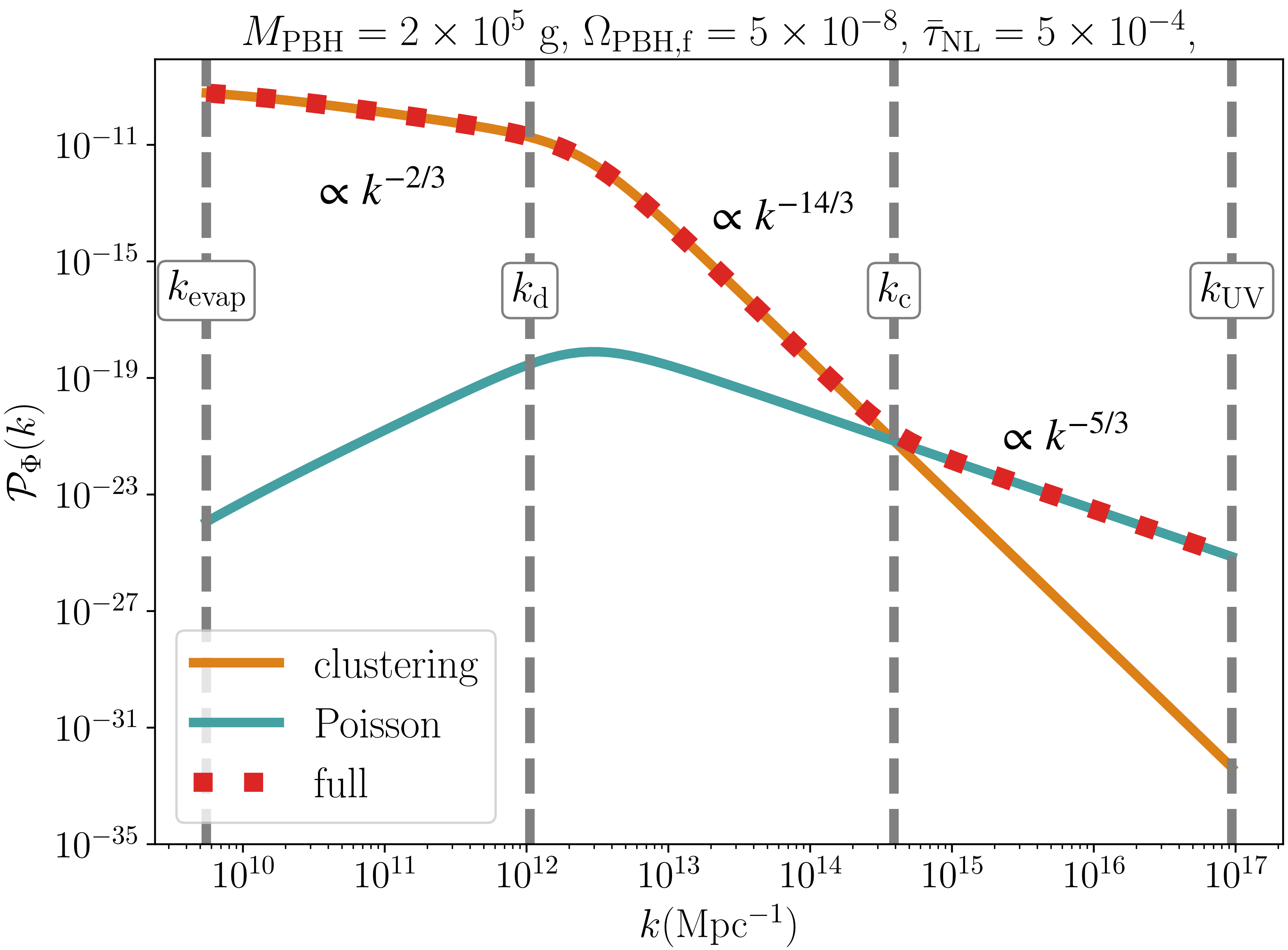
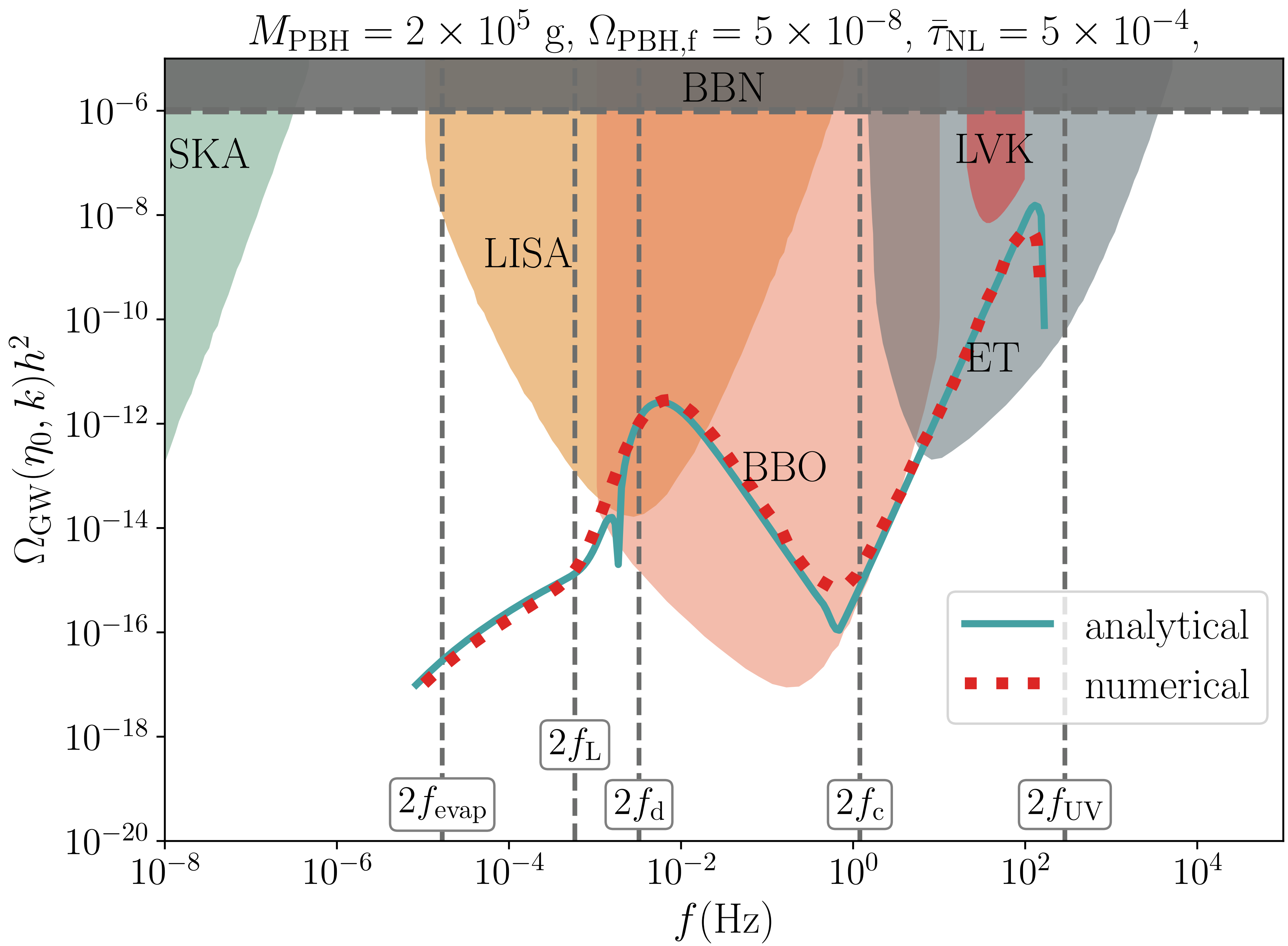


Scale Hierarchy : $10^5\mathrm{Mpc}^{-1} < k_{\rm evap} < k_{\rm d} < k_{\rm c} < k_{\rm UV} \ll k_{\rm f} \sim 1/R$

Non-Gaussian Induced GWs



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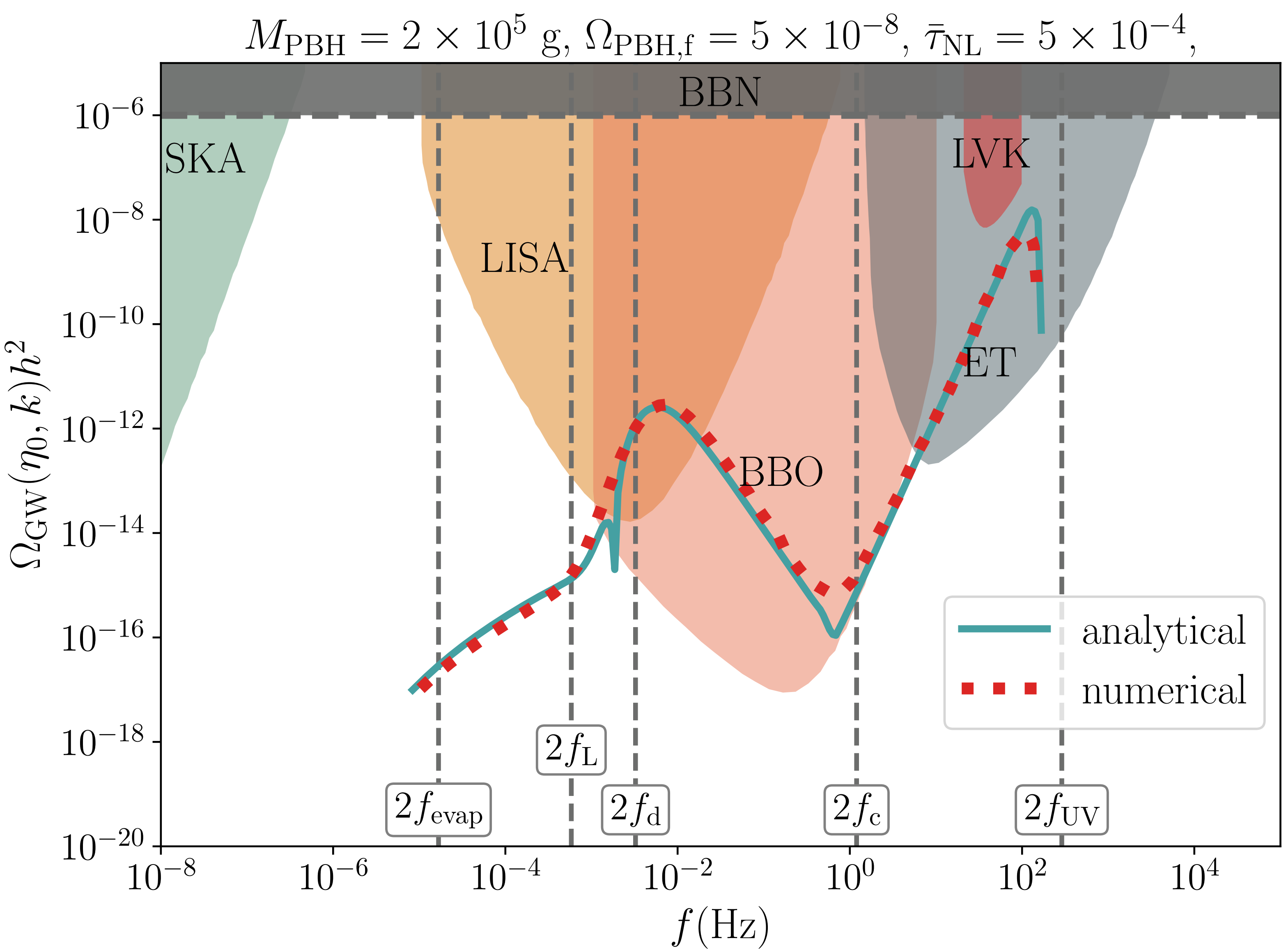
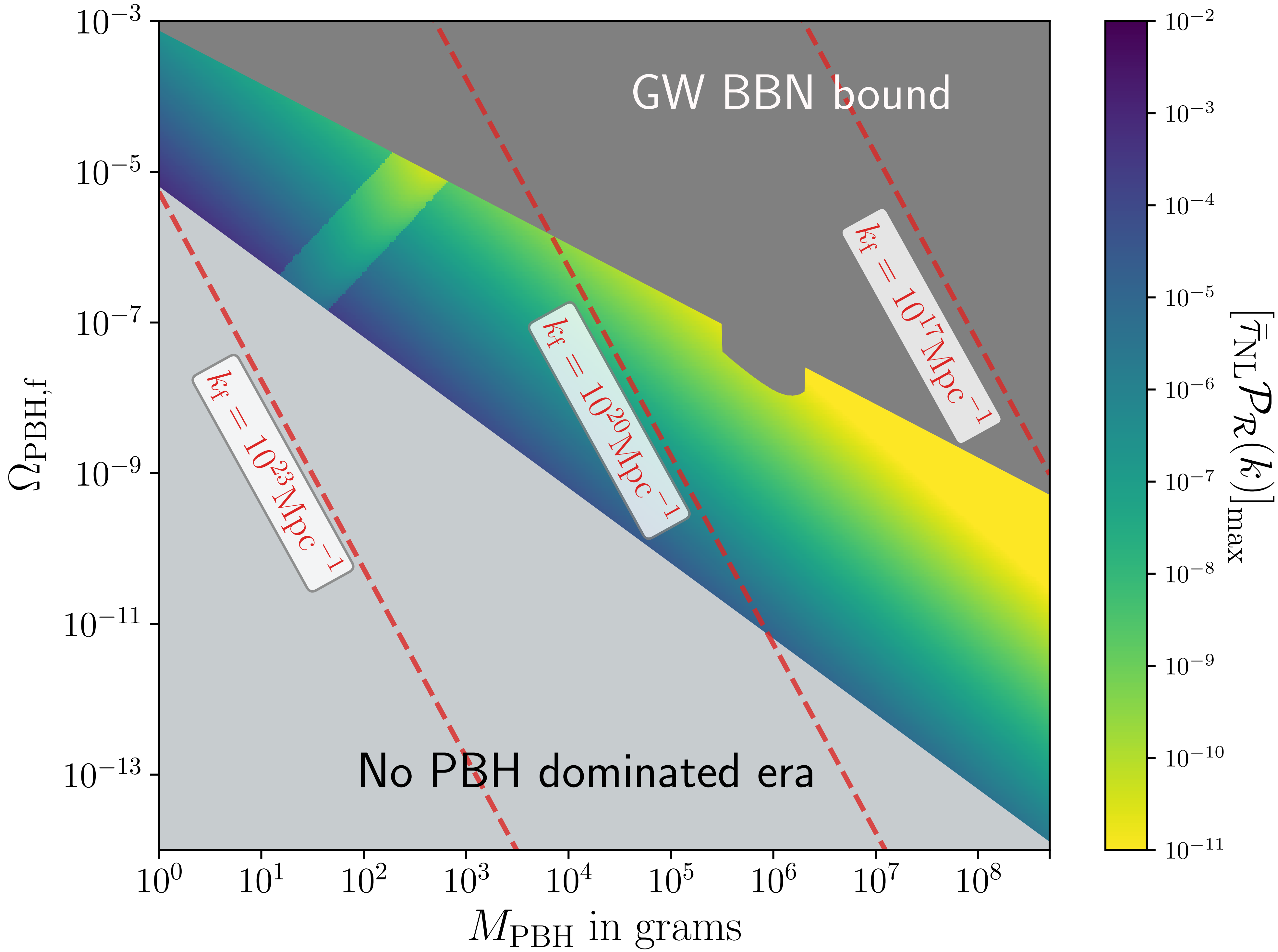


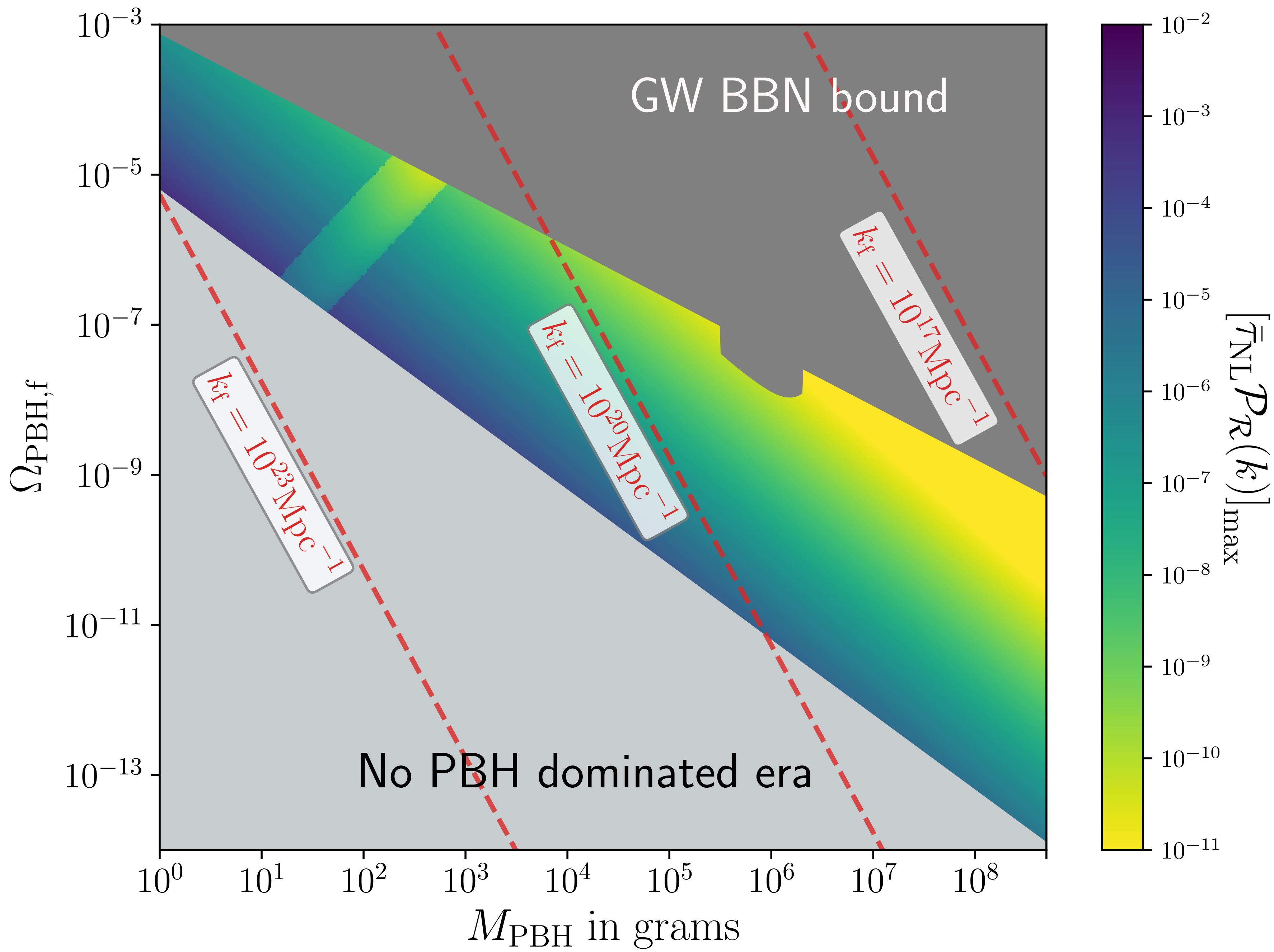
$$\Omega_{\rm GW}(\eta_0,k)h^2 \simeq \begin{cases} 3\times10^{-80}\left(\frac{k}{10^4{\rm Mpc}^{-1}}\right)^{11/3}\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{41/6}\left(\frac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{16/3} & 2k_{\rm c}<k<2k_{\rm UV} \\ 5\times10^{-13}\left(\frac{k}{10^4{\rm Mpc}^{-1}}\right)^{-7/3}\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{11/6}\left(\frac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{16/3}\left(\frac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & 2k_{\rm d}<k<2k_{\rm c} \\ 4\times10^{-75}\left(\frac{k}{10^4{\rm Mpc}^{-1}}\right)^{17/3}\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{17/2}\left(\frac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & \\ \quad +10^{-39}\left(\frac{k}{10^4{\rm Mpc}^{-1}}\right)\left(\frac{M_{\rm PBH}}{10^4{\rm g}}\right)^{4}\left(\frac{\Omega_{\rm PBH,f}}{10^{-10}}\right)^{22/9}\left(\frac{\bar{\tau}_{\rm NL}\mathcal{P}_{\mathcal{R}}}{10^{-15}}\right)^{2} & 2k_{\rm evap}<k<2k_{\rm d} \end{cases}$$

Constraining non-Gausianities



Constraining non-Gausianities

$$\Omega_{\rm GW}(2k_{\rm d},\eta_0) \leq 10^{-6} \Rightarrow \bar{\tau}_{\rm NL}\mathscr{P}_{\mathscr{R}}(k) \leq 4\times 10^{-20}\Omega_{\rm PBH,f}^{-17/9}\left(\frac{M_{\rm PBH}}{10^4 {\rm g}}\right)^{-17/9}$$



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- The portal of **PBH induced GWs induced** can serve as a **new messenger from the early Universe.**

Thanks for your attention!

Appendix